Candidates should be aware that high marks will only be awarded for answers to numerical problems if all the algebraic steps in the calculation are shown. A correct numerical answer without working will not score high marks.

Question 1

a) Differentiate the following expressions with respect to the variable, simplifying your answers as far as possible:

(i) $y = 3 \sin 2\theta - 5 \tan \theta$ (ii) $y = \cos^3 x$ (iii) $y = e^{4m} \cos 3m$ (iv) $y = \dfrac{\log_e x}{x^2}$

(10 marks)

b) The distance s [m] moved by a body in a time t [s] is given by:

$$s = 2t^3 - 9t^2 + 12t + 6 \ .$$

Determine the velocity and acceleration when (a) t = 0 [s]
(b) t = 3 [s] **(10 marks)**

Question 2

a) An open storage tank of height h [m] with a square base of side x [m] is to be constructed so that it has a volume of 500 [m^3].

(i) Show that the surface area of the base and four walls (A) will be,

$$A = x^2 + \frac{2000}{x} \quad [m^2]$$

(ii) Find the value of x such that the expression for A is a minimum.

(10 marks)

b) An open cylindrical storage tank of height h [m] and base radius r [m] is to be constructed, from sheet metal with no overlaps, so that it has a volume of 2 [m^3].

(a) Show that the height h can be expressed as, $h = \dfrac{2}{\pi r^2}$ [m]

(b) Find the value of r such that the amount of sheet metal used is a minimum.

(10 marks)

Question 3

a) Determine the following indefinite integrals:

(i) $\int (2x^2 + 3x + 2)\,dx$ (ii) $\int (\sin 7\theta + 2\cos 5\theta)\,d\theta$ **(6 marks)**

b) Evaluate the following definite integrals, correct to 3 decimal places:

(i) $\int_0^{\frac{\pi}{2}} (\cos\theta + \sin\theta)\,d\theta$ (ii) $\int_1^2 (2x^3 - x^2)\,dx$ **(6 marks)**

c) Find the area bounded by the curve $y = 3x^2 + 2x + 3$ the x axis and the lines $x = 1$ and $x = 4$. correct to 3 decimal places.

(8 marks)

Question 4

a) Find the particular solution of the following first order differential equations,

(i) $\frac{d\theta}{dt} = 2e^{(3t-2\theta)}$, if $\theta = 0$ when $t = 0$

(ii) $\frac{dy}{dx} + \frac{y}{x} = -1$, if $y = 1$ when $x = 1$ **(10 marks)**

b) The angular velocity ($d\theta/dt$) of a point on a linkage is given by.

$$2t\left(t - \frac{d\theta}{dt}\right) = 5$$, where θ is the angular displacement at time t [s].

By separating the variables determine an expression for the angular displacement θ [rad], given that when t= 1 [s], θ = 0.5 [rad].

(10 marks)

Question 5

a) The displacement x [m] of a point on a critically damped body mounted on a spring in terms of the time t [s] from the original displacement is given by;

$$9\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 4x = 0,$$

Solve this equation for x given that $x = 3$ and $\frac{dx}{dt} = 4$ when $t = 0$ **(10 marks)**

b) The equation of motion of a body oscillating on the end of a spring is given by,

$$\frac{d^2s}{dt^2} + 225s = 0$$

Where s [m] is the displacement from the origin, after time t [s].

Solve the equation for s given that $x = 1$ and $\frac{ds}{dt} = 0$ when $t = 0$

(10 marks)

NOTE

For linear second order differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0,$

If the roots of the auxiliary equation are **real and different** say m = α and m = β then the general solution is of the form: $y = A\,e^{\alpha x} + B\,e^{\beta x}$

If the roots of the auxiliary equation are **real and equal** say m = α twice then the general solution is of the form: $y = (Ax + B)\,e^{\alpha x}$

If the roots of the auxiliary equation are **complex** say m = $\alpha \pm j\beta$ then the general solution is of the form: $y = e^{\alpha x}(C\cos\beta x + D\sin\beta x)$